## 5.6 – Logarithmic Functions

*Daily Objectives:*

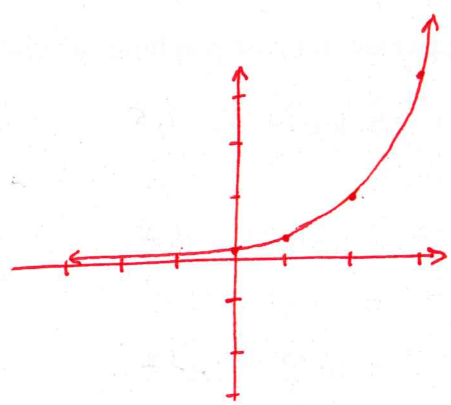
1.Review solving equations for exponents
2. Define logarithm with base b and common logarithm with base 10.
3. Discover, using a calculator, that $\log(10^x) = x$.
4. Solve logarithmic equations with base 10 and with bases other than 10.
5. Establish that the inverse of an exponential function is a logarithmic function.
6. Use change-of-base property.

**Investigation: Exponents and Logarithms**

You will discover the connections between exponents with base 10 and logarithms.

**Step 1:** Graph the function $f(x)=10^x$ for $-1.5 \le x \le 1.5$ on your calculator and sketch the graph below:

| | |
|---|---|
| Domain | All real numbers |
| Range | $y > 0$ |
| x - intercept | None |
| y – intercept | 1 |
| Equation of asymptote | $y = 0$ |



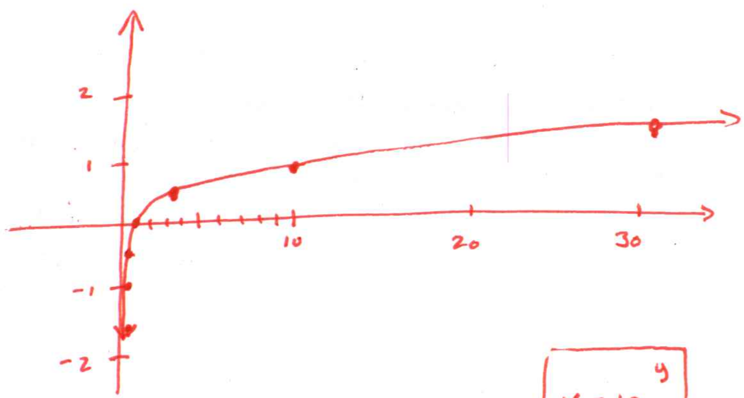
**Step 2:** Complete the table of values for $f(x)=10^x$ and its inverse:

| $x$ | -1.5 | -1 | -0.5 | 0 | 0.5 | 1 | 1.5 |
|---|---|---|---|---|---|---|---|
| f($x$) | .0316 | .1 | .316 | 1 | 3.16 | 10 | 31.6 |

| $x$ | .0316 | .1 | .316 | 1 | 3.16 | 10 | 31.6 |
|---|---|---|---|---|---|---|---|
| f($x$) | -1.5 | -1 | -0.5 | 0 | 0.5 | 1 | 1.5 |

**Step 3:** Plot the points for the inverse on your calculator. Notice that it has the properties below:

| | |
|---|---|
| Domain | $x > 0$ |
| Range | all real numbers |
| x - intercept | 1 |
| y – intercept | None |
| Equation of asymptote | $x = 0$ |




$\boxed{x = 10^y}$

Inverse of:

$y = 10^x$

**Step 4:** This <u>inverse function</u> is called the **<u>logarithm of *x* or log(x)</u>**. Enter the equation $y = \log x$ into your calculator and graph.

**Step 5:** Find the following values on your graphing calculator.

a. $10^{1.5} =$ 31.6 b. $\log(10^{1.5}) =$ 1.5 c. $\log 0.32 =$ -.49

d. $10^{1.2} =$ 15.85 e. $\log(10^{1.2}) =$ 1.2 f. $10^{\log 2.8} =$ 2.8

g. $\log 10^{2.8} =$ 2.8 h. $10^{\log 0.32} =$ .32

**Step 6:** Based on your results from Step 5, what is $\log 10^x$? x

**Step 7:** Based on your results from Step 5, what is $10^{\log x}$? x

**Definition of Logarithm**

For $a > 0$ and $b > 0$, $\log_b a = x$ is equivalent to $a = b^x$.

**Logarithm Change-of-Base Property**

$$\log_b a = \frac{\log a}{\log b} \text{ where } a > 0 \text{ and } b > 0$$

**Use the definitions above to write the expressions below a different way**

*a.* $\log 10{,}000 = 4$

$\log_{10}$

$10^4 = 10{,}000$

*b.* $\log 18 = x$

$\log_{10}$

$10^x = 18$

*c.* $\log_4 16 = 2$

$4^2 = 16$

*d.* $\log_9 100 = x$

$9^x = 100$

*e.* $3^x = 10$

$\log_3 10 = x$

*f.* $19^2 = 361$

$\log_{19} 361 = 2$

*g.* $\log_8 13 = x$

$8^x = 13$

*h.* $\log_{15} 36 = x$

$15^x = 36$

***When the exponent of an equation is a variable, use logarithms to solve.***

***When the variable is inside of a logarithm, change the equation into exponential form to solve.***

**Solve the equations below:**

$\log_3 10 = x$

$\frac{\log_{10} 10}{\log_{10} 3} = x$ ← Change of base formula

$\boxed{2.096 \approx x}$

$3^x = 234$

$\log_3 234 = x$

$\frac{\log_{10} 234}{\log_{10} 3} = x$

$\boxed{4.966 \approx x}$

$\log_x 27 = 3$

$x^3 = 27$

$(x^3)^{\frac{1}{3}} = (27)^{\frac{1}{3}}$

$\boxed{x = 3}$

$17^2 = x$

$289 = x$

$\log_4 x = 5$

$4^5 = x$

$1024 = x$

$x^5 = 2567$

$(x^5)^{\frac{1}{5}} = (2567)^{\frac{1}{5}}$

$x = 4.807$

**Example**: An initial deposit of $500 is invested at 8.5% interest, compounded annually. How long will it take until the balance grows to $800?

$$y = 500(1+.085)^x$$

$$800 = 500(1.085)^x$$